STUDY ON THE SIERPINSKI AND RIESEL NUMBERS

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Abstract

In this paper we examine in detail and in depth the Sierpinski and Riesel numbers.

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1. SIERPIŃSKI NUMBER

A Sierpinski number is an odd positive number k such that all integers of the form $k \cdot 2^n + 1$ are composite for each natural number *n ≥ 1*, or for *N*+.

In other words, when k is a Sierpinski number, all the elements of this set are composite:

$$\{ k2^n + 1 : n \in \mathbb{N} \}_+$$

There is an infinite number of odd integers that, used in place of k, and that do not produce prime numbers and are so Sierpinski numbers.

First, we note that k can only be odd and not even.
If it were even or is a power of 2 and then merges in 2^n or is a even number composite which in part merges with 2^n and the factor that remains becomes an odd number, and then return to the case that k is odd.

For example chosen 6 we have:

$3\ 2^{n+1} + 1 \rightarrow 3\ 2^n + 1$

The first 29 Sierpinski numbers that are currently known:

78557, 271129, 271577, 322523, 327739, 482719, 575041, 603713, 903983, 934909, 965431, 1259779, 1290677, 1518781, 1624097, 1639459, 1777613, 2131043, 2131099, 2191531, 2510177, 2541601, 2576089, 2931767, 2931991, 3083723, 3098059, 3555593, 3608251, ….

Some of these numbers are also prime numbers (ie: 271129, 322523, 327739, 482719, 934909, 1639459, 2131043, 2131099, 2576089, 3098059, 3608251…)

1.1 THE SIERPINSKI NUMBER 78557

For example, let's consider the first of these numbers, the number composite 78557

78557 2^n + 1

All numbers that are derived from this formula with n ≥ 1 have at least a factor in a set of numbers {3, 5, 7, 13, 19, 37, 73}.

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:
3: every 2n, or any even exponent is divisible by at least 3
5: each 4n + 1 is divisible by at least for 5
7: every 3n + 1 is divisible by at least for 7
13: every 12n + 11 is divisible by at least 13
19: every 18n + 15 is divisible by at least 19
37: every 36n + 27 is divisible by at least 37
73: every 9n + 3 is divisible by at least 73

It's easy to demonstrate that these numbers, for example the 3 is divisible every 2n because:
78 557 has as the sum of digits 5
Multiplied for n even we have:
2^2 = 4, 2^4 = 16 (=7), 2^6 = 64 (=1), (da 256 (=4), 1024 (=7) e 4096 (=1) are repeated each 3 times, we obtain

5*4 + 1 = 21 (=3)
5*16 + 1 = 81 (=0)

5*64 + 1 = 321 (=6)

And then it is shown that

78557 2^{2n} + 1 is divisible by 3

The set of these numbers {3, 5, 7, 13, 19, 37, 73} entirely covers all the numbers *n ε N+*
For all the exponents n odd there is at least a factor (for n even already know that it is always divisible by 3), and then with 7 numbers belonging to the set is covered the whole N+

For the first 100 values of odd integers n we have:
n = 1 (divisors /5/7), 3 (/73), 5 (/5), 7 (/7), 9 (/5), 11 (/13), 13 (/5/7), 15 (/19), 17 (/5), 19 (/7), 21 (/5/73), 23 (/13), 25 (/5/7), 27 (/37), 29 (/5), 31 (/7), 33 (/5/19), 35 (/13), 37 (/5/7), 39 (/73), 41 (/5), 43 (/7), 45 (/5), 47 (/13), 49 (/5/7), 51 (/19), 53 (/5), 55 (/7), 57 (/5/73), 59 (/13), 61 (/5/7), 63 (/37), 65 (/5), 67 (/7), 69 (/5/19), 71 (/13), 73 (/5/7), 75 (/73), 77 (/5), 79 (/7), 81 (/5), 83 (/13), 85 (/5/7), 87 (/19), 89 (/5), 91 (/7), 93 (/5/73), 95 (/13), 97 (/5/7), 99 (/37), 101 (/5), 103 (/7), 105 (/5/19)

1	157115	5 7 67^2
2	314229	3 104743
3	628457	73 8609
4	1256913	3^2 7 71 281
5	2513825	5^2 193 521
6	5027649	3 11 131 1163
7	10055297	7 1436471
8	20110593	3 541 12391
9	40221185	5 59 136343
10	80442369	3^3 7^2 41 1483
11	160884737	13 523 23663
12	321769473	3 43 47 73 727
13	643538945	5 7 1759 10453
14	1287077889	3 353 599 2029
15	2574155777	19 135481883
16	5148311553	3^2 7 11 7429021
17	10296623105	5 2059324621
18	20593246209	3 6864415403
19	41186492417	7 1583 3716857
20	82372984833	3 53 173 311 9629
21	1,64746E+11	5 73 451358821
22	3,29492E+11	3^2 7 101 51782483
23	6,58984E+11	13 811 62504399
24	1,31797E+12	3 439322585771
25	2,63594E+12	5^3 7 47563 63337
26	5,27187E+12	3 11 29 43 128110399
27	1,05437E+13	37 167 40427 42209
28	2,10875E+13	3^3 7 1873 59569669
29	4,2175E+13	5 75659 111486983
30	8,43499E+13	3 41 73 859 10936129

31	**1,687E+14**	**7^2 109 31585821557**
32	**3,374E+14**	**3 463^2 524640139**
33	**6,74799E+14**	**5 19 541301 13122371**
34	**1,3496E+15**	**3 7**
35	**2,6992E+15**	**13**
36	**5,3984E+15**	**3**
37	**1,07968E+16**	**5 7**
38	**2,15936E+16**	**3**
39	**4,31872E+16**	**73**
40	**8,63743E+16**	**3 7**
41	**1,72749E+17**	**5**
42	**3,45497E+17**	**3**
43	**6,90995E+17**	**7**
44	**1,38199E+18**	**3**
45	**2,76398E+18**	**5**
46	**5,52796E+18**	**3 7**
47	**1,10559E+19**	**13**
48	**2,21118E+19**	**3 73**
49	**4,42237E+19**	**5 7**
50	**8,84473E+19**	**3**

1.2 THE SIERPINSKI NUMBER 271129

Let's consider the number 271129

$271129\ 2^n + 1$

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:

3: every 2n + 1, or any odd exponent is divisible by at least 3
5: each 4n is divisible by at least 5
7: every 3n + 2 is divisible by at least 7
13: every 12n + 6 is divisible by at least 13
17: every 8n + 6 is divisible by at least 17
241: every 24n + 10 is divisible by at least 241

For the first 100 values of *n*, it is sufficient to consider only the even exponent:
n = 2 (divisors /7), 4 (/5), 6 (/13/17), 8 (/5/7), 10 (/241), 12 (/5), 14 (/7/17), 16 (/5), 18 (/13), 20 (/5/7), 22 (/17), 24 (/5), 26 (/7), 28 (/5), 30 (/13/17), 32 (/5/7), 34 (/241), 36 (/5), 38 (/7/17), 40 (/5), 42 (/13), 44 (/5/7), 46 (/17), 48 (/5), 50 (/7), 52 (/5), 54 (/13/17), 56 (/5/7), 58 (/241), 60 (/5), 62 (/7/17), 64 (/5), 66 (/13), 68 (/5/7), 70 (/17), 72 (/5), 74 (/7), 76 (/5), 78 (/13/17), 80 (/5/7), 82 (/241), 84 (/5), 86 (/7/17), 88 (/5), 90 (/13), 92 (/5/7), 94 (/17), 96 (/5), 98 (/7), 100 (/5), 102 (/13/17), 104 (/5/7), 106 (/241),

Filling the entire set of natural even numbers 2n, with only 6 factors, we have that the number 271129 is a Sierpinski number.

1	**542259**	**3^2 60251**
2	**1084517**	**7^2 22133**
3	**2169033**	**3 127 5693**
4	**4338065**	**5 37 131 179**
5	**8676129**	**3 7 11 23^2 71**
6	**17352257**	**13 17 78517**
7	**34704513**	**3^2 419 9203**
8	**69409025**	**5^2 7 396623**
9	**138818049**	**3 139 463 719**
10	**277636097**	**47 127 193 241**
11	**555272193**	**3 7 29 911777**
12	**1110544385**	**5 222108877**
13	**2221088769**	**3^9 112843**
14	**4442177537**	**7 17 37329223**
15	**8884355073**	**3 11 53 313 16229**
16	**17768710145**	**5 23 154510523**
17	**35537420289**	**3 7 127 13324867**
18	**71074840577**	**13 19 41 7018351**
19	**142149681153**	**3^2 107453 146989**
20	**284299362305**	**5 7 8122838923**
21	**568598724609**	**3 311 609430573**
22	**1137197449217**	**17 66893967601**
23	**2274394898433**	**3 7^2 15472074139**
24	**4548789796865**	**5 109 127 6967 9433**
25	**9097579593729**	**3^2 11 22013 4174567**
26	**18195159187457**	**7 1031 29759 84719**
27	**36390318374913**	**3 23 853 618283609**
28	**72780636749825**	**5^2 2911225469993**
29	**145561273499649**	**3 7 6931489214269**
30	**291122546999297**	**13 17 40847 32249531**

31	582245093998593	3^3 127 457 371554181
32	1,16449E+15	5 7
33	2,32898E+15	3
34	4,65796E+15	59 241 327587084323
35	9,31592E+15	3 7
36	1,86318E+16	5
37	3,72637E+16	3
38	7,45274E+16	7 17
39	1,49055E+17	3
40	2,98109E+17	5
41	5,96219E+17	3 7
42	1,19244E+18	13
43	2,38488E+18	3
44	4,76975E+18	5 7
45	9,5395E+18	3
46	1,9079E+19	17
47	3,8158E+19	3 7
48	7,6316E+19	5
49	1,52632E+20	3
50	3,05264E+20	7

1.3 COVERING SETS OF SIERPINSKI NUMBERS

For these numbers, there is a limited set of numbers covering the entire space of exponents n ε N+
Furthermore, all the Sierpinski numbers have covering sets similar.

n	**covering set**
78557	**{3, 5, 7, 13, 19, 37, 73}**
271129	**{3, 5, 7, 13, 17, 241}**
271577	**{3, 5, 7, 13, 17, 241}**
322523	**{3, 5, 7, 13, 37, 73, 109}**
327739	**{3, 5, 7, 13, 17, 97, 257}**
482719	**{3, 5, 7, 13, 17, 241}**
575041	**{3, 5, 7, 13, 17, 241}**
603713	**{3, 5, 7, 13, 17, 241}**
903983	**{3, 5, 7, 13, 17, 241}**
934909	**{3, 5, 7, 13, 19, 73, 109}**
965431	**{3, 5, 7, 13, 17, 241}**
1259779	**{3, 5, 7, 13, 19, 73, 109}**
1290677	**{3, 5, 7, 13, 19, 37, 109}**
1518781	**{3, 5, 7, 13, 17, 241}**
1624097	**{3, 5, 7, 13, 17, 241}**
1639459	**{3, 5, 7, 13, 17, 241}**
1777613	**{3, 5, 7, 13, 17, 19, 109, 433}**
2131043	**{3, 5, 7, 13, 17, 241}**

1.4 PROOF THAT SETS COVERING THE ENTIRE SPACE OF EXPONENTS n ε N+

For Sierpinski numbers we have a set of prime numbers that will divide any member of the sequence, so called because it is said to "cover" that sequence.

Here is the proof for the numbers 78557, 271129

78557 has a covering set formed by

{3, 5, 7, 13, 19, 37, 73}

They are divisible by:

3: every 2n, or any even exponent is divisible by at least 3
5: each 4n + 1 is divisible at least for 5
7: every 3n + 1 is divisible at least for 7
13: every 12n + 11 is divisible least for 13
19: every 18n + 15 is divisible least for 19
37: every 36n + 27 is divisible least for 37
73: every 9n + 3 is divisible least for 73

We need to consider only the odd exponents:

5 is repeated every 4 times and 7 is repeated every 6 times for odd numbers. But every 12 times give rise to the same number, let's see in detail:

5: 1, 5, 9, 13, 17, 21, 25, …

7: 1, 4, 7, 10, 13, 16, 19, 22, 25,

13 and 25 are repeated, so we have to count them only once!

$\frac{1}{4} + \frac{1}{6} - \frac{1}{12} = \frac{1}{3}$

To demonstrate the effectiveness of the set we must arrive at a value of $\frac{1}{2}$ to have all the odd exponents *n* (even *n* already worth $\frac{1}{2}$ with the divisor 3)

Applying for the other remaining divisors you get:

13: $\frac{1}{12}$
19: $\frac{1}{18}$
37: $\frac{1}{36}$
73: $\frac{1}{18}$ **because even *n* we have to take them off**

Since we also have the repetitions we have to subtract:

for the divisors 5 e 19: $\frac{1}{36}$

for the divisors 5 e 73: $\frac{1}{36}$

Now we add and subtract:

$\frac{1}{4} + \frac{1}{6} + \frac{1}{12} + \frac{1}{18} + \frac{1}{36} + \frac{1}{18} - \frac{1}{12} - \frac{1}{36} - \frac{1}{36} = \frac{1}{2}$

CVD

For the Sierpinski number 271129 by applying the same reasoning has:

271129 has a covering set formed by

{3, 5, 7, 13, 17, 241}

They are divisible by:

3: every 2n + 1, or any odd exponent is divisible by at least 3
5: each 4n is divisible by at least 5
7: every 3n + 2 is divisible at least for 7
13: every 12n + 6 is divisible by at least 13
17: every 8n + 6 is divisible by at least 17
241: every 24n + 10 is divisible at least 241

We need to consider only the even exponents:

5: $\frac{1}{4}$
7: $\frac{1}{6}$
13: $\frac{1}{12}$
17: $\frac{1}{8}$
241: 1/24

for the divisors 5 e 7: 1/12
for the divisors 7 e 17: 1/24
for the divisors 13 e 17: 1/24

Now we add and subtract:

$$\frac{1}{4} + \frac{1}{6} + \frac{1}{12} + \frac{1}{8} + \frac{1}{24} - \frac{1}{12} - \frac{1}{24} - \frac{1}{24} = \frac{1}{2}$$

CVD

We note that there is the number 24, that is related to the modes corresponding to the physical vibrations of the bosonic strings by the following Ramanujan function:

$$24 = \frac{4\left[anti\log\frac{\int_0^\infty \frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2 w'}dx}{e^{-\frac{\pi t^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2 w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}.$$

1.5 SIERPINSKI PROBLEM AND VERIFICATION OF THE LAST 6 NUMBERS OF CANDIDATES TO BE SIERPINSKI NUMBERS

The problem is to find which is the Sierpinski number smaller in absolute.

It is conjectured that it is precisely the smallest absolute 78557.

To proof this it's started a massive distributed computing project with super-computer to see if all the odd numbers k < 78557 could be Sierpinski numbers and that for each of these there is a exponent *n* such that

$k2^n+1$ is a prime number.

According to what said before, we can see if there is a limited set of numbers covering the entire space of exponents *n* ε N+.
As of February 2013, there are only six candidates that are the following:

k = 10223, 21181, 22699, 24737, 55459, e 67607

1.5.1 THE CANDIDATE NUMBER 10223

Let's consider 10223, which is also a prime number.

$10223\ 2^n + 1$

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:

3: every 2n, or any even exponent is divisible by at least 3
5: each 4n + 3 is divisible by at least 5
7: each 3n + 1 is divisible by at least 7
11: every 10n + 3 is divisible by at least 11
13: every 12n + 9 is divisible by at least 13
23: each 11n + 1 is divisible by at least 23
67: every 66n + 41 is divisible by at least 67
127: every 7n + 1 is divisible by at least 127
277: every 276n + 5 is divisible by at least 277
673: every 48n + 17 is divisible by at least 673
619033: every 619032n + 77 is divisible by at least 619033
45677096693: each 45677096692n + 101 is divisible by at least 45677096693

For the first 100 values of *n*, it is sufficient to consider only the odd exponent:
n = 1 (divisors /7/23/127), 3 (/5/11), 5 (/277),7 (/5/7), 9 (/13), 11 (/5), 13 (/7/11), 15 (/127), 17 (/673), 19 (/5/7), 21 (/13), 23 (/5/11/23), 25 (/7), 27 (/5), 29 (/127), 31 (/5/7), 33 (/11/13), 35 (/5), 37 (/7), 39 (/5), 41 (/67), 43 (/5/7/11/127), 45 (/13/23), 47 (/5), 49 (/7), 51 (/5), 53 (/11), 55 (/5/7), 57

(/13/127), 59 (/5), 61 (/7), 63 (/5/11), 65 (/673), 67 (/5/7/23), 69 (/13), 71 (/5/127), 73 (/7/11), 75 (/5), 77 (/619033), 79 (/5/7), 81 (/13), 83 (/5/11), 85 (/7/127), 87 (/5), 89 (/23), 91 (/5/7), 93 (/11/13), 95 (/5), 97 (/7), 99 (/5/127), 101 (/45677096693), 103 (/5/7/11), 105 (/13), 107 (/67)

If only for the first hundred values of the exponent *n* requires too many factors (exactly 12) and too high as 619033 and 45677096693.
For *n* = 43 there are 4 divisors (5, 7, 11, 127), for other values of n there are three, and they must be at most two.
Not filling the entire set of odd natural numbers 2n + 1 we have that surely the number

$10223\ 2^n + 1$

becomes a prime number for some value *n*.

We can conclude that 10223 is not a Sierpinski number, without resorting to a huge computational and out of our reach.
But with this method relatively simple we are able to verify and determine which are the possible numbers of Sierpinski.

1	**20447**	**7 23 127**
2	**40893**	**3 43 317**
3	**81785**	**5 11 1487**
4	**163569**	**3 7 7789**
5	**327137**	**277 1181**
6	**654273**	**3^2 139 523**
7	**1308545**	**5 74 109**
8	**2617089**	**3 127 6869**
9	**5234177**	**13 19 21191**
10	**10468353**	**3 7 498493**
11	**20936705**	**5 773 5417**
12	**41873409**	**3^3 23 67429**
13	**83746817**	**7 11 1087621**
14	**167493633**	**3 6257 8923**
15	**334987265**	**5 29 127 18191**
16	**669974529**	**3 7 43 61 12163**
17	**1339949057**	**673 997 1997**
18	**2679898113**	**3^2 11177 26641**
19	**5359796225**	**5^2 7 113 131 2069**
20	**10719592449**	**3 107 2143 15583**
21	**21439184897**	**13 1649168069**
22	**42878369793**	**3 7 127 16077379**
23	**85756739585**	**5 11^2 23 1667 3697**
24	**1,71513E+11**	**3^2 37 397 1297369**
25	**3,43027E+11**	**7 59 830573749**
26	**6,86054E+11**	**3 228684638891**
27	**1,37211E+12**	**5 19 149 1283 75553**
28	**2,74422E+12**	**3 7^2 18668133787**
29	**5,48843E+12**	**127 211 2083 98327**
30	**1,09769E+13**	**3^3 43 3659 2583947**

31	**2,19537E+13**	**5 7 491 7001 182473**
32	**4,39075E+13**	**3 83 16763 10519307**
33	**8,78149E+13**	**11 13^2 47237709163**
34	**1,7563E+14**	**3 7 23 53 10837 633091**
35	**3,5126E+14**	**5 71 1039 952323077**
36	**7,02519E+14**	**3^2 101 127 6085420603**
37	**1,40504E+15**	**7 2146003 93531917**
38	**2,81008E+15**	**3 3469 21013 12850043**
39	**5,62015E+15**	**5^2 13445893 16719317**
40	**1,12403E+16**	**3 7 5352522731940669**
41	**2,24806E+16**	**67 26633 12598339027**
42	**4,49612E+16**	**3^2 4995692164779577**
43	**8,99225E+16**	**5 7 11 29 109 127 581807383**
44	**1,79845E+17**	**3 43 9829 141840131309**
45	**3,5969E+17**	**13 19 23 17041 3715423297**
46	**7,1938E+17**	**3 7 1210103 28308478571**
47	**1,43876E+18**	**5 113 685271 3716014123**
48	**2,87752E+18**	**3^5 4817 2458301990219**
49	**5,75504E+18**	**7^2 617 5701 10243 3259783**
50	**1,15101E+19**	**3 127 14106667 2141552639**

1.5.2 THE CANDIDATE NUMBER 21181

Let's consider 21181

21181 2^n + 1

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:

3: every 2n + 1, ie every odd exponent is divisible by at least 3
5: each 4n + 2 is divisible by at least 5
7: every 3n is divisible by at least 7
13: every 12n + 4 is divisible by at least 13
17: every 8n is divisible by at least 17
89: 11n is divisible by at least 11
157: every 156n + 92 is divisible by at least 157
83077: Each 83077n + 20 is divisible by at least 83077
342467: every 342466n + 68 is divisible by at least 342467

For the first 100 values of *n*, it is sufficient to consider only the even exponent:
n = 2 (divisors /5), 4 (/13), 6 (/5/7), 8 (/17), 10 (/5), 12 (/7), 14 (/5), 16 (/13/17), 18 (/5/7), 20 (/83077), 22 (/5/89), 24 (/7/17), 26 (/5), 28 (/13), 30 (/5/7), 32 (/17), 34 (/5), 36 (/7), 38 (/5), 40 (/13/17), 42 (/5/7), 44 (/89), 46 (/5), 48 (/7/17), 50 (/5), 52 (/13), 54 (/5/7), 56 (/17), 58 (/5), 60 (/7), 62 (/5), 64 (/13/17), 66 (/5/7/89), 68 (/342467), 70 (/5), 72 (/7/17), 74 (/5), 76 (/13), 78 (/5/7), 80 (/17), 82 (/5), 84 (/7), 86 (/5), 88 (/13/17/89), 90 (/5/7), 92 (/157), 94 (/5), 96 (/7/17), 98 (/5), 100 (/13), 102 (/5/7), 104 (/17), 106 (/5), 108 (/7)

If only for the first hundred values of the exponent *n* requires factors too high as 83077 and 342467.
For *n* = 66 and 89 there are 3 divisors and they must be at most two.
Not filling the entire set of even natural numbers 2n we have that surely the number

21181 2^n + 1

becomes a prime number for some value *n*.

We can conclude that 21181 is not a Sierpinski number.

1	42363	3^4 523
2	84725	5^2 3389
3	169449	3 7 8069
4	338897	13 131 199
5	677793	3 225931
6	1355585	5 7^2 11 503
7	2711169	3^2 301241
8	5422337	17 467 683
9	10844673	3 7 101 5113
10	21689345	5 23 188603
11	43378689	3 37 89 4391
12	86757377	7 941 13171
13	173514753	3^2 19279417
14	347029505	5 6469 10729
15	694059009	3 7 53 71 8783
16	1388118017	11 13 17 19 41 733
17	2776236033	3 292 739 1489
18	5552472065	5 7 158642059
19	11104944129	3^3 47 8750941
20	22209888257	83077 267341
21	44419776513	3 7 23 67 1372633
22	88839553025	5^2 89 139 287251
23	1,77679E+11	3 59226368683
24	3,55358E+11	7 17 2986203463
25	7,10716E+11	3^2 7681 10281017
26	1,42143E+12	5 11 25844233607
27	2,84287E+12	3 73 773 1187 3011
28	5,68573E+12	13 633091 690839
29	1,13715E+13	3 5651 670764041

30	**2,27429E+13**	**5 7 683 951387809**
31	**4,54859E+13**	**3^2 514933 9814837**
32	**9,09717E+13**	**17 23 3607 64503521**
33	**1,81943E+14**	**3 7 89 7823 122443819**
34	**3,63887E+14**	**5 19 271 5107 2767627**
35	**7,27774E+14**	**3 223 1087852942261**
36	**1,45555E+15**	**7 11 41 421 10211 107251**
37	**2,91109E+15**	**3^3 83 2707 479872859**
38	**5,82219E+15**	**5 1164437789396173**
39	**1,16444E+16**	**3 7 55449418546749**
40	**2,32888E+16**	**13 17 61 97 157 269 421697**
41	**4,65775E+16**	**3 89459 173552545769**
42	**9,3155E+16**	**5^2 7 47 5749 25423 77491**
43	**1,8631E+17**	**3^2 23 23509 38285275123**
44	**3,7262E+17**	**89 353 2887237 4107893**
45	**7,4524E+17**	**3 7 29 293 787 5306847647**
46	**1,49048E+18**	**5 11 4919491 5508627437**
47	**2,98096E+18**	**3 37 103340057 259875047**
48	**5,96192E+18**	**7^2 17^2 3181 132351436757**
49	**1,19238E+19**	**3^2 167 2851 2782659389141**
50	**2,38477E+19**	**5 71 67176580075587659**

1.5.3 *THE CANDIDATE NUMBER* 22699

Let's consider 22699, which is also a prime number

22699 2^n + 1

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:

3: every 2n + 1, ie every odd exponent is divisible by at least 3
5: each 4n is divisible by at least 5
7: each 3n + 2 is divisible by at least 7
11: every 10n + 6 is divisible by at least 11
13: every 12n + 6 is divisible by at least 13
17: every 8n + 2 is divisible by at least 17
19: every 18n + 4 is divisible by at least 19
53: every 52n + 14 is divisible by at least 53
73: every 9n + 7 is divisible by at least 73
84884846681: each 84884846680n +190 is divisible by at least 84884846681

For the first 100 values of *n*, it is sufficient to consider only the even exponent:
n = 2 (divisors /7/17), 4 (/5/19), 6 (/11/13), 8 (/5/7), 10 (/17), 12 (/5), 14 (/7/53), 16 (/5/11/73), 18 (/13/17), 20 (/5/7), 22 (/19), 24 (/5), 26 (/7/11/17), 28 (/5), 30 (/13), 32 (/5/7), 34 (/17/73), 36 (/5/11), 38 (/7), 40 (/5/19), 42 (/13/17), 44 (/5/7), 46 (/11), 48 (/5), 50 (/7/17), 52 (/5/73), 54 (/13), 56 (/5/7/11), 58 (/17/19), 60 (/5), 62 (/7), 64 (/5), 66 (/11/13/17/53), 68 (/5/7), 70 (/73), 72 (/5), 74 (/7/17), 76 (/5/11/19), 78 (/13), 80 (/5/7), 82 (/17), 84

(/5), 86 (/7/11), 88 (/5/73), 90 (/13/17), 92 (/5/7), 94 (/19), 96 (/5/11), 98 (/7/17), 100 (/5), 102 (/13), 104 (/5/7), 106 (/11/17/73), 108 (/5), 110 (/7), 112 (/5/19), 114 (13/17), 116 (/5/7/11), 118 (/53),… 190 (/84884846681)

If only for the first hundred values of the exponent *n* requires factors too high as 84884846681
For different values of *n* there are 3 divisors and they must be at most two.
Not filling the entire set of even natural numbers 2n we have that surely the number

22699 2^n + 1

becomes a prime number for some value *n*.

We can conclude that 22699 is not a Sierpinski number.

1	**45399**	**3 37 409**
2	**90797**	**7^2 17 109**
3	**181593**	**3^2 20177**
4	**363185**	**5 19 3823**
5	**726369**	**3 7 34589**
6	**1452737**	**11 13 10159**
7	**2905473**	**3 73 13267**
8	**5810945**	**5 7 166027**
9	**11621889**	**3^2 1291321**
10	**23243777**	**17 23 59447**
11	**46487553**	**3 7 83 149 179**
12	**92975105**	**5 18595021**
13	**185950209**	**3 431 143813**
14	**371900417**	**7 53 1002427**
15	**743800833**	**3^3 1259 21881**
16	**1487601665**	**5 11 73 370511**
17	**2975203329**	**3 7 113 233 5381**
18	**5950406657**	**13 17 26924917**
19	**11900813313**	**3 10133 391487**
20	**23801626625**	**5^3 7 2293 11863**
21	**47603253249**	**3^2 23 9973 23059**
22	**95206506497**	**19 47 1721 61949**
23	**1,90413E+11**	**3 7^2 1295326619**
24	**3,80826E+11**	**5 97 785208301**
25	**7,61652E+11**	**3 29 73 277 563 769**
26	**1,5233E+12**	**7 11 17 1163715893**
27	**3,04661E+12**	**3^2 213307 1586971**
28	**6,09322E+12**	**5 67961 17931509**
29	**1,21864E+13**	**3 7 373 1555781033**
30	**2,43729E+13**	**13 173 63841 169753**

31	4,87457E+13	3 16248577108651
32	9,74915E+13	5 7 23 269 6991 64399
33	1,94983E+14	3^5 101 7944543263
34	3,89966E+14	17 73 2711 115911167
35	7,79932E+14	3 7 1579 60161 390967
36	1,55986E+15	5 11 28361152771463
37	3,11973E+15	3 37 193 443 887 370603
38	6,23945E+15	7 109 3539 2310688801
39	1,24789E+16	3^2 857 33331 48540571
40	2,49578E+16	5^2 19 42961 1223034083
41	4,99156E+16	3 7 61 38966142761729
42	9,98313E+16	13 17 67 107 24443 2577871
43	1,99663E+17	3 23 73 659 60150490471
44	3,99325E+17	5 7^2 11789681 138247853
45	7,9865E+17	3^2 47 103 113 22877 7090901
46	1,5973E+18	11 233 1213 5507 6329 14741
47	3,1946E+18	3 7 152123821341790013
48	6,3892E+18	5 250998263 5091031643
49	1,27784E+19	3 131 14888557 2183892989
50	2,55568E+19	7 17 8269 25972069403107

1.5.4 *THE CANDIDATE NUMBER* 24737

Let's consider 24737

24737 2^n + 1

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:

3: every 2n, or any even exponent is divisible by at least 3
5: each 4n + 1 is divisible by at least 5
7: every 3n is divisible least per7
11: every 10n + 9 is divisible by at least 11
13: every 12n + 11 is divisible by at least 13
17: every 8n + 3 is divisible by at least 17
31: 5n is divisible by at least 5
503: every 502n + 31 is divisible by at least 503
907: every 906n + 7 is divisible by at least 907
267133: every 267132n + 151 is divisible by at least 267133
2118089: each 2118088n +103 is divisible by at least 2118089
13736837: each 13736836n +127 is divisible by at least 13736837

For the first 100 values of *n*, it is sufficient to consider only the odd exponent:
n = 1 (divisors /5), 3 (/7/17), 5 (/5/31),7 (/907), 9 (/5/7/11), 11 (/13/17), 13 (/5), 15 (/7/31), 17 (/5), 19 (/11/17), 21 (/5/7), 23 (/13), 25 (/5/31), 27 (/7/17), 29 (/5/11), 31 (/503), 33 (/5/7), 35 (/13/17), 37 (/5), 39 (/7/11), 41 (/5), 43 (/17), 45 (/5/7/31), 47 (/13), 49 (/5/11), 51 (/7/17), 53 (/5), 55 (/31), 57 (/5/7), 59 (/11/13/17), 61 (/5), 63 (/7), 65 (/5/31), 67 (/17), 69 (/5/7/11), 71 (/13), 73

(/5), 75 (/7/17/31), 77 (/5), 79 (/11), 81 (/5/7), 83 (/13/17), 85 (/5/31), 87 (/7), 89 (/5/11), 91 (/17), 93 (/5/7), 95 (/13/31), 97 (/5), 99 (/7/11/17), 101 (/5), 103 (/2118089),… 127 (/13736837),… 151 (/267133)

If only for the first hundred values of the exponent *n* requires factors too high as 267133, 2118089 and 13736837
For different values of *n* there are 3 divisors and they must be at most two.
Not filling the entire set of odd natural numbers 2n+1 we have that surely the number

$24737\ 2^n + 1$

becomes a prime number for some value *n*.

We can conclude that 24737 is not a Sierpinski number.

1	**49475**	**5^2 1979**
2	**98949**	**3 32983**
3	**197897**	**7 17 1663**
4	**395793**	**3^3 107 137**
5	**791585**	**5 31 5107**
6	**1583169**	**3 7 75389**
7	**3166337**	**907 3491**
8	**6332673**	**3 2110891**
9	**12665345**	**5 7 11 67 491**
10	**25330689**	**3^2 31 163 557**
11	**50661377**	**13 17 229237**
12	**101322753**	**3 7 4824893**
13	**202645505**	**5 1609 25189**
14	**405291009**	**3 135097003**
15	**810582017**	**7 31 3735401**
16	**1621164033**	**3^2 180129337**
17	**3242328065**	**5 648465613**
18	**6484656129**	**3 7^3 19 47 7057**
19	**12969312257**	**11 17^2 4079683**
20	**25938624513**	**3 31 1741 160201**
21	**51877249025**	**5^2 7 296441423**
22	**1,03754E+11**	**3^5 1097 389219**
23	**2,07509E+11**	**13 97 599 274723**
24	**4,15018E+11**	**3 7 4943 3998131**
25	**8,30036E+11**	**5 31 59 61 1487933**
26	**1,66007E+12**	**3 677 817366799**
27	**3,32014E+12**	**7 17 991 1451 19403**
28	**6,64029E+12**	**3^2 263 4231 663049**
29	**1,32806E+13**	**5 11 107747 2241037**
30	**2,65612E+13**	**3 7 31 40800539939**

31	**5,31223E+13**	**503 12689 8323031**
32	**1,06245E+14**	**3 37 957158612623**
33	**2,12489E+14**	**5 7 223 821 33160481**
34	**4,24978E+14**	**3^2 47219824889401**
35	**8,49957E+14**	**13^2 17 31 83873 113783**
36	**1,69991E+15**	**3 7**
37	**3,39983E+15**	**5**
38	**6,79965E+15**	**3**
39	**1,35993E+16**	**7 11**
40	**2,71986E+16**	**3 31**
41	**5,43972E+16**	**5**
42	**1,08794E+17**	**3 7**
43	**2,17589E+17**	**17**
44	**4,35178E+17**	**3**
45	**8,70356E+17**	**5 7 31**
46	**1,74071E+18**	**3**
47	**3,48142E+18**	**13**
48	**6,96285E+18**	**3 7**
49	**1,39257E+19**	**5 11**
50	**2,78514E+19**	**3 31**

1.5.5 *THE CANDIDATE NUMBER* 55459

Let's consider 55459

$55459 \cdot 2^n + 1$

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:

3: every 2n + 1, ie every odd exponent is divisible by at least 3
5: each 4n is divisible by at least 5
7: every 3n + 2 is divisible by at least 7
11: every 10n + 2 is divisible by at least 11
13: every 12n + 6 is divisible by at least 13
37: every 36n + 34 is divisible by at least 37
43: every 14n + 2 is divisible by at least 43
47: every 23n is divisible by at least 47
181: every 180n + 10 is divisible by at least 181
613: every 612n +154 is divisible by at least 613
138230459: each 138230458n +130 is divisible by at least 138230459

For the first 100 values of *n*, it is sufficient to consider only the even exponent:
n = 2 (divisors /7/11/43), 4 (/5), 6 13), 8 (/5/7), 10 (/181), 12 (/5/11), 14 (/7), 16 (/5/43), 18 (/13), 20 (/5/7), 22 (/11), 24 (/5), 26 (/7), 28 (/5), 30 (/13/43), 32 (/5/7/11), 34 (/37), 36 (/5), 38 (/7), 40 (/5), 42 (/11/13), 44 (/5/7/43), 46 (/47), 48 (/5), 50 (/7), 52 (/5/11), 54 (/13), 56 (/5/7), 58 (/43), 60 (/5), 62 (/7/11), 64 (/5), 66 (/13), 68 (/5/7), 70 (/37), 72 (/5/11/43), 74 (/7), 76 (/5), 78 (/13), 80 (/5/7), 82 (/11), 84 (/5), 86 (/7/43), 88 (/5), 90 (/13), 92 (/5/7/11/47),

94 (/), 96 (/5), 98 (/7), 100 (/5/43), 102 (/11/13), 104 (/5/7), 106 (/37), 108 (/5), 110 (/7), 112 (/5/11), 114 (13/43), 116 (/5/7),…130 (/138230459),…. , 154 (/613)

If only for the first hundred values of the exponent *n* requires factors too high as 138230459
For different values of *n* there are 3 or 4 divisors and they must be at most two.
Not filling the entire set of odd natural numbers 2n we have that surely the number

$55459\ 2^n + 1$

becomes a prime number for some value *n*.

We can conclude that 55459 is not a Sierpinski number.

1	110919	3 36973
2	221837	7 11 43 67
3	443673	3^2 49297
4	887345	5 103 1723
5	1774689	3 7 84509
6	3549377	13 273029
7	7098753	3 61 38791
8	14197505	5 7^2 167 347
9	28395009	3^3 173 6079
10	56790017	181 211 1487
11	113580033	3 7 5408573
12	227160065	5 11 223 18521
13	454320129	3 3083 49121
14	908640257	7 1229 105619
15	1817280513	3^2 12689 15913
16	3634561025	5^2 43 3380987
17	7269122049	3 7 19 29 628219
18	14538244097	13^2 199 432287
19	29076488193	3 4129 2347339
20	58152976385	5 7 461 3604151
21	116305952769	3^2 4073 3172817
22	232611905537	11 709 1151 25913
23	465223811073	3 7 47 797 591407
24	930447622145	5 83 2242042463
25	1860895244289	3 101 307 373 53633
26	3721790488577	7 531684355511
27	7443580977153	3^3 547 504000337
28	14887161954305	5 2977432390861
29	29774323908609	3 7^3 28935203021
30	59548647817217	13 43 40853 2607571

31	**119097295634433**	**3 97 16183 25290061**
32	**238194591268865**	**5 7 11^3 5113117769**
33	**476389182537729**	**3^2 52932131393081**
34	**952778365075457**	**37 25750766623661**
35	**1,90556E+15**	**3 7**
36	**3,81111E+15**	**5**
37	**7,62223E+15**	**3**
38	**1,52445E+16**	**7**
39	**3,04889E+16**	**3**
40	**6,09778E+16**	**5**
41	**1,21956E+17**	**3 7**
42	**2,43911E+17**	**11 13**
43	**4,87823E+17**	**3**
44	**9,75645E+17**	**5 7 43**
45	**1,95129E+18**	**3**
46	**3,90258E+18**	**47 19477 208889 20408747**
47	**7,80516E+18**	**3 7**
48	**1,56103E+19**	**5**
49	**3,12206E+19**	**3**
50	**6,24413E+19**	**7**

1.5.6 *THE CANDIDATE NUMBER* 67607

Let's consider 67607

67607 2^n + 1

All these numbers always end with the digit 3, 5, 7 or 9.

They are divisible by:

3: every 2n, or any even exponent is divisible by at least 3
5: each 4n + 1 is divisible by at least 5
11: every 10n + 5 is divisible by at least 11
13: every 12n + 7 is divisible by at least 13
17: every 8n + 7 is divisible by at least 17
19: every 18n + 11 is divisible by at least 19
31: every 5n + 3 is divisible by at least 31
41: every 40n + 19 is divisible by at least 41
43: every 14n + 9 is divisible by at least 43
73: every 9n + 3 is divisible by at least 73
198017: each 198016n +27 is divisible by at least 198017
1236173: each 1236172n +131 is divisible by at least 1236173

For the first 100 values of *n*, it is sufficient to consider only the odd exponent:
n = 1 (divisors /5), 3 (/31/73), 5 (/5/11),7 (/13/17), 9 (/5/43), 11 (/19), 13 (/5/31), 15 (/11/17), 17 (/5), 19 (/13/41), 21 (/5/73), 23 (/17/31/43), 25 (/5/11), 27 (/198017), 29 (/5/19), 31 (/13/17), 33 (/5/31), 35 (/11), 37 (/5/43), 39 (/17/73), 41 (/5), 43 (/13/31), 45 (/5/11), 47 (/17/19), 49 (/5), 51 (/43), 53 (/5/31), 55 (/11/13/17), 57 (/5/73), 59 (/41), 61 (/5), 63 (/17/31), 65

(/5/11/19/43), 67 (/13), 69 (/5), 71 (/17), 73 (/5/31), 75 (/11/73), 77 (/5), 79 (/13/17/43), 81 (/5), 83 (/19/31), 85 (/5/11), 87 (/17), 89 (/5), 91 (/13), 93 (/5/31/43/73), 95 (/11/17), 97 (/5), 99 (/41), 101 (/5/19), 103 (/13/17/31),…131 (/1236173)

If only for the first hundred values of the exponent *n* requires factors too high as 198017 and 1236173
For different values of *n* there are 3 or 4 divisors and they must be at most two.
Not filling the entire set of odd natural numbers 2n + 1 we have that surely the number

$67607\ 2^n + 1$

becomes a prime number for some value *n*.

We can conclude that 67607 is not a Sierpinski number.

1	135215	5 27043
2	270429	3 109 827
3	540857	31 73 239
4	1081713	3 23 61 257
5	2163425	5^2 11 7867
6	4326849	3^2 480761
7	8653697	13 17 39157
8	17307393	3 31 149 1249
9	34614785	5 43 131 1229
10	69229569	3 23076523
11	138459137	19 29 251287
12	276918273	3^2 73 521 809
13	553836545	5 31 3573139
14	1107673089	3 4483 82361
15	2215346177	11 17 23 37 13921
16	4430692353	3 9283 159097
17	8861384705	5 331 5354311
18	17722769409	3^3 31 277 76441
19	35445538817	13 41 66501949
20	70891077633	3 257 91946923
21	141782155265	5 53 73 1493 4909
22	283564310529	3 139 23027 29531
23	567128621057	17 31 43 25026637
24	1134257242113	3^2 126028582457
25	2268514484225	5^2 11 179 1871 24631
26	4537028968449	3 23 28163 2334767
27	9074057936897	198017 45824641
28	18148115873793	3 31 195141030901
29	36296231747585	5 19 1423 7757 34613

30	72592463495169	3^2 73 15427 7162171
31	145184926990337	13 17 67 9805154791
32	290369853980673	3 1087 89043193493
33	580739707961345	5 31 283 13239250153
34	1,16148E+15	3
35	2,32296E+15	11
36	4,64592E+15	3
37	9,29184E+15	5 43
38	1,85837E+16	3 31
39	3,71673E+16	17 73
40	7,43347E+16	3
41	1,48669E+17	5
42	2,97339E+17	3
43	5,94677E+17	13 31
44	1,18935E+18	3
45	2,37871E+18	5 11
46	4,75742E+18	3
47	9,51484E+18	17 19
48	1,90297E+19	3 31 73
49	3,80594E+19	5
50	7,61187E+19	3

1.6 CONCLUSIONS

We have seen that the 6 possible candidates are not Sierpinski numbers.

The reasons are as follows:

- **For the first hundred values of the exponent *n* factors requires too high**
- **The set of numbers already for the first hundred values is greater than 8 elements**
- **For different values of *n* there are 3 or 4 divisors and they must be at most two.**

The third condition is the strongest and this is enough to prove whether some odd number k is a Sierpinski number.

A very important observation concerns that these numbers of the set

$$k\ 2^n + 1$$

always end with the digit 3, 5, 7 or 9.

The primality test just do so only with numbers ending with the digit 7 because those ending in digits 3 and 9 are always divisible by 3 and those ending with the digit 5 are obviously divisible by 5.
This means that you just have to test every 4 numbers of the set.

Besides if we wanted to apply the coating proof of the covering sets as we did for the Sierpinski numbers 78557 and 271129 we will never achieve the value of $\frac{1}{2}$

2. RIESEL NUMBER

A Riesel number is an odd positive number k such that all integers of the form $k \cdot 2^n - 1$ are composite for each natural number *n ≥ 1,* or for *N*+.

In other words, when k is a Riesel number, all the elements of this set are composite:

$$\{ k \cdot 2^n - 1 : n \in \mathbb{N} \}_+$$

There are infinitely many integers k such that $k \cdot 2^n - 1$ it is not prime for any integer *n*.

The number 509203 has this property, and the same applies to the numbers in the form

$$509203 + 11184810 \cdot k; k \in \mathbb{N}.$$

Here, as for the Sierpinski numbers, to prove that a number is a Riesel number, we need to find a "*set covering*".
A set covering is a set of small primes such that every member of a certain sequence is divisible by one of them, and is so named because it is said that "covers" the succession.
The only proven Riesel numbers smaller than a million have the following covering sets:

- **$509203 \cdot 2^n - 1$ has covering set {3, 5, 7, 13, 17, 241}**
- **$762701 \cdot 2^n - 1$ has covering set {3, 5, 7, 13, 17, 241}**
- **$777149 \cdot 2^n - 1$ has covering set {3, 5, 7, 13, 19, 37, 73}**

- **790841*2^n-1 has covering set {3, 5, 7, 13, 19, 37, 73}**
- **992077*2^n-1 has covering set {3, 5, 7, 13, 17, 241}**

2.1 THE RIESEL PROBLEM

The Riesel problem is to find the smallest Riesel number.

It hasn't found any covering set for values of k < 509203, it is conjectured that this is the smallest Riesel number.

Currently the 10 smaller candidates <509203 are the following

2293, 9221, 23669, 31859, 38473, 40597, 46663, 67117, 74699, 81041.

2.1.1 THE CANDIDATE NUMBER 2293

Let's consider 2293

$2293\ 2^n - 1$

All these numbers always end with the digit 1, 3, 5 or 7.

They are divisible by:

3: every 2n, or any even exponent is divisible by at least 3
5: each 4n + 1 is divisible by at least 5
7: each 3n + 1 is divisible by at least 7
13: every 12n + 3 is divisible by at least 13
17: every 8n + 3 is divisible by at least 17
23: every 11n + 7 is divisible by at least 23
941: every 940n +71 is divisible by at least 941
2017: every 2016n +23 is divisible by at least 2017
19913: every 19912 +47 is divisible by at least 19913

For the first 100 values of *n*, it is sufficient to consider only the odd exponent:
n = 1 (divisors /5/7), 3 (/13/17), 5 (/5), 7 (/7/23), 9 (/5), 11 (/17), 13 (/5/7), 15 (/13), 17 (/5), 19 (/7/17), 21 (/5), 23 (/2017), 25 (/5/7), 27 (/13/17), 29 (/5/23), 31 (/7), 33 (/5), 35 (/17), 37 (/5/7), 39 (/13), 41 (/5), 43 (/7/17), 45 (/5), 47 (/19913), 49 (/5/7), 51 (/13/17/23), 53 (/5), 55 (/7), 57 (/5), 59 (/17), 61 (/5/7), 63 (/13), 65 (/5), 67 (/7/17), 69 (/5), 71 (/941), 73 (/5/7/23), 75 (/13/17), 77 (/5), 79 (/7), 81 (/5), 83 (/17), 85 (/5/7), 87 (/13), 89 (/5), 91 (/7/17), 93 (/5), 95 (/23), 97 (/5/7), 99 (/13/17), 101 (/5), 103 (/7)

For the first hundred values of the exponent *n* requires factors set too high, as 2017 and 19913.
For *n* = 51 there are 3 divisors and they must be at most two.
Not filling the entire set of odd natural numbers 2n + 1 we have that surely the number

$2293\ 2^n - 1$

becomes a prime number for some value *n*.

We can conclude that 2293 is not a Riesel number.

1	**4585**	**5 7 131**
2	**9171**	**3^2 1019**
3	**18343**	**13 17 83**
4	**36687**	**3 7 1747**
5	**73375**	**5^3 587**
6	**146751**	**3 11 4447**
7	**293503**	**7 23 1823**
8	**587007**	**3^4 7247**
9	**1174015**	**5 234803**
10	**2348031**	**3 7^2 15973**
11	**4696063**	**17 276239**
12	**9392127**	**3 67 46727**
13	**18784255**	**5 7 19 47 601**
14	**37568511**	**3^2 307 13597**
15	**75137023**	**13 193 29947**
16	**150274047**	**3 7 11 650537**
17	**300548095**	**5 5407 11117**
18	**601096191**	**3 23 37 235447**
19	**1202192383**	**7 17 1669 6053**
20	**2404384767**	**3^2 503 531121**
21	**4808769535**	**5 733 1312079**
22	**9617539071**	**3 7 283 593 2729**
23	**19235078143**	**2017 9536479**
24	**38470156287**	**3 3457 3709397**
25	**76940312575**	**5^2 7 439658929**
26	**153880625151**	**3^3 11 59 8781637**
27	**307761250303**	**13^3 17 29 149 1907**
28	**615522500607**	**3 7 29310595267**
29	**1231045001215**	**5 23 199 53792659**
30	**2462090002431**	**3 271 4297 704771**

31	4924180004863	7^2 19 5289129973
32	9848360009727	3^2 22853 47882651
33	19696720019455	5 3939344003891
34	39393440038911	3 7 109 53993 318743
35	78786880077823	17^2 613 444729139
36	157573760155647	3 11 47 967 105061991
37	315147520311295	5 7 53 169890846529
38	630295040622591	3^2 79 601 1475026481
39	1,26059E+15	13
40	2,52118E+15	3 7 23
41	5,04236E+15	5
42	1,00847E+16	3
43	2,01694E+16	7 17
44	4,03389E+16	3
45	8,06778E+16	5
46	1,61356E+17	3 7
47	3,22711E+17	19913 693409 23371559
48	6,45422E+17	3
49	1,29084E+18	5 7
50	2,58169E+18	3

2.1.2 THE CANDIDATE NUMBER 9221

Let's consider 9221

9221 2^n - 1

All these numbers always end with the digit 1, 3, 5 or 7.

They are divisible by:

3: every 2n +1, or any even exponent is divisible by at least 3
5: each 4n is divisible by at least 5
7: every 3n + 2 is divisible by at least 7
11: every 10n + 2 is divisible by at least 11
13: every 12n + 10 is divisible by at least 13
47: every 23n + 8 is divisible by at least 47
53: every 52n + 26 is divisible by at least 53
59: every 58n + 18 is divisible by at least 59
101: every 100n + 6 is divisible by at least 101
211: every 210n + 90 is divisible by at least 211
4513: every 4512n +30 is divisible by at least 4513
1874073577: each 1874073576n +66 is divisible at least for 1874073577

For the first 100 values of *n*, it is sufficient to consider only the even exponent:
n = 2 (divisors /7/11), 4 (/5), 6 (/101), 8 (/5/7/47), 10 (/13), 12 (/5/11), 14 (/7), 16 (/5), 18 (/59), 20 (/5/7), 22 (/11/13), 24 (/5), 26 (/7/53), 28 (/5), 30 (/4513), 32 (/5/7/11), 34 (/13), 36 (/5), 38 (/7), 40 (/5), 42 (/11), 44 (/5/7), 46 (/13), 48 (/5), 50 (/7), 52 (/5/11), 54 (/47), 56 (/5/7), 58 (/13), 60 (/5), 62 (/7/11), 64 (/5), 66 (/1874073577), 68 (/5/7), 70 (/13), 72 (/5/11), 74 (/7), 76

(/5/59), 78 (/53), 80 (/5/7), 82 (/11/13), 84 (/5), 86 (/7), 88 (/5), 90 (/211), 92 (/5/7/11), 94 (/13), 96 (/5), 98 (/7), 100 (/5/47), 102 (/11), 104 (/5/7), 106 (/13/101), 108 (/5), 110 (/7), 112 (/5/11)

For the first hundred values of the exponent *n* requires factors set too high, as 4513 and 1874073577.
For different values of *n* there are 3 divisors and they must be at most two.

Not filling the entire set of even natural numbers 2n we have that surely the number

$9221\ 2^n - 1$

becomes a prime number for some value *n*.

We can conclude that 9221 is not a Riesel number.

1	18441	3^3 683
2	36883	7 11 479
3	73767	3 67 367
4	147535	5 19 1553
5	295071	3 7 14051
6	590143	101 5843
7	1180287	3^2 131143
8	2360575	5^2 7^2 41 47
9	4721151	3 1573717
10	9442303	13 726331
11	18884607	3 7 269 3343
12	37769215	5 11 347 1979
13	75538431	3^2 2221 3779
14	151076863	7 29 744221
15	302153727	3 131^2 5869
16	604307455	5 120861491
17	1208614911	3 7 57553091
18	2417229823	59 40969997
19	4834459647	3^4 59684687
20	9668919295	5 7 229 1206353
21	19337838591	3 3593 1794029
22	38675677183	11 13 19 1361 10459
23	77351354367	3 7 683 1447 3727
24	154702708735	5 30940541747
25	309405417471	3^2 2999 11463281
26	618810834943	7 53 16921 98573
27	1237621669887	3 47119 8755291
28	2475243339775	5^2 41 97 107 232669
29	4950486679551	3 7^2 359 1381 67927
30	9900973359103	4513 2193878431

31	19801946718207	3^2 47 2143 2477 8819
32	39603893436415	5 7 11 383 268582913
33	79207786872831	3 37 713583665521
34	158415573745663	13 557 21877582343
35	316831147491327	3 7 15087197499587
36	633662294982655	5 193 38971 16849577
37	1,26732E+15	3
38	2,53465E+15	7
39	5,0693E+15	3
40	1,01386E+16	5
41	2,02772E+16	3 7
42	4,05544E+16	11 29 127129739432257
43	8,11088E+16	3
44	1,62218E+17	5 7
45	3,24435E+17	3
46	6,4887E+17	13
47	1,29774E+18	3 7 829 645907 115410149
48	2,59548E+18	5
49	5,19096E+18	3
50	1,03819E+19	7

2.1.3 *THE CANDIDATE NUMBER* 23669

Let's consider *23669*

23669 2^n - 1

All these numbers always end with the digit 1, 3, 5 or 7.

They are divisible by:

3: every 2n + 1, or any even exponent is divisible by at least 3
5: each 4n + 2 is divisible by at least 5
7: each 3n + 2 is divisible by at least 7
13: every 12n + 4 is divisible by at least 13
31: each 5n + 1 is divisible by at least 31
37: every 36n + 24 is divisible by at least 37
97: every 48n is divisible by at least 97
199: every 99N + 12 is divisible by at least 199
751: Each 750 + 84 is divisible by at least 751
1409 every 1408 + 72 is divisible by at least 1409

For the first 100 values of *n*, it is sufficient to consider only the even exponent:
n = 2 (divisors /5/7), 4 (/13), 6 (/5/31), 8 (/7), 10 (/5), 12 (/199), 14 (/5/7), 16 (/13/31), 18 (/5), 20 (/7), 22 (/5), 24 (/37), 26 (/5/7/31), 28 (/13), 30 (/5), 32 (/7), 34 (/5), 36 (/31), 38 (/5/7), 40 (/13), 42 (/5), 44 (/7), 46 (/5/31), 48 (/97), 50 (/5/7), 52 (/13), 54 (/5), 56 (/7/31), 58 (/5), 60 (/37), 62 (/5/7), 64 (/13), 66 (/5/31), 68 (/7), 70 (/5), 72 (/1409), 74 (/5/7), 76 (/13/31), 78 (/5), 80 (/7), 82 (/5), 84 (/751), 86 (/5/7/31), 88 (/13), 90 (/5), 92 (/7), 94 (/5), 96 (/31/37/97), 98 (/5/7), 100 (/13), 102 (/5), 104 (/7), 106 (/5/31),

For the first hundred values of the exponent *n* requires factors set too high, as 751 and 1409.
For different values of *n* there are 3 divisors and they must be at most two.

Not filling the entire set of even natural numbers 2n we have that surely the number

$23669\ 2^n - 1$

becomes a prime number for some value *n*.

We can conclude that 23669 is not a Riesel number.

1	**47337**	**3 31 509**
2	**94675**	**5^2 7 541**
3	**189351**	**3^3 7013**
4	**378703**	**13 29131**
5	**757407**	**3 7 36067**
6	**1514815**	**5 29 31 337**
7	**3029631**	**3 11 91807**
8	**6059263**	**7 865609**
9	**12118527**	**3^2 47 28649**
10	**24237055**	**5 23 419 503**
11	**48474111**	**3 7 19 31 3919**
12	**96948223**	**199 487177**
13	**193896447**	**3 64632149**
14	**387792895**	**5 7 11079797**
15	**775585791**	**3^2 2539 33941**
16	**1551171583**	**13 31 3849061**
17	**3102343167**	**3 7 11 1801 7457**
18	**6204686335**	**5 181 523 13109**
19	**12409372671**	**3 53^2 1472573**
20	**24818745343**	**7^2 506505007**
21	**49637490687**	**3^5 23 31 286493**
22	**99274981375**	**5^3 6247 127133**
23	**198549962751**	**3 7 179 619 85331**
24	**397099925503**	**37^2 290065687**
25	**794199851007**	**3 269 971 1013531**
26	**1588399702015**	**5 7 31 131 11175289**
27	**3176799404031**	**3^2 11 337 9011 10567**
28	**6353598808063**	**13 103 821 5779577**
29	**12707197616127**	**3 7 19 137 6547 35507**
30	**25414395232255**	**5 5082879046451**

31	50828790464511	3 31 546546134027
32	101657580929023	7 23 47 109 2423 50867
33	203315161858047	3^2 139 569 4261 67033
34	406630323716095	5 29 409 21839 313961
35	813260647432191	3 7 38726697496771
36	1,62652E+15	31 1873 28013042641
37	3,25304E+15	3
38	6,50609E+15	5 7
39	1,30122E+16	3
40	2,60243E+16	13
41	5,20487E+16	3 7 31
42	1,04097E+17	5^2 499 1801 3527 1313651
43	2,08195E+17	3
44	4,16389E+17	7
45	8,32779E+17	3
46	1,66556E+18	5 31
47	3,33112E+18	3 7
48	6,66223E+18	97 337 33617 6062602751
49	1,33245E+19	3
50	2,66489E+19	5 7

2.2 CONCLUSIONS

A very important observation concerns that these numbers of the set

$k\ 2^n - 1$

always end with the digit 1, 3, 5 or 7.

The primality test just do so only with numbers ending with the digit 3 because those ending in digits 1 and 7 are always divisible by 3 and those ending with the digit 5 are obviously divisible by 5.
This means that you just have to test every 4 numbers of the set.

Besides if we wanted to apply the coating proof of the covering sets as we did for the Sierpinski numbers 78557 and 271129 we will never achieve the value of $\frac{1}{2}$

3. CURIOSITY ABOUT SIERPINSKI AND RIESEL NUMBERS

The related issues include, of course, possible smaller Sierpinski and Riesel numbers. Maybe it will solve them with this work, but our observations on their relationships with the forms arithmetic of prime numbers 6k ± 1 (except 2 and 3 initials) will open the door to a subsequent proof.

All the numbers of the set $k \cdot 2^n + 1$ are composite for every natural integer *n* if k is a Sierpinski number.

The same applies to the Riesel numbers of the set $k \cdot 2^n - 1$

This means that when k is a Sierpinski or Riesel number,the result of the respective formulas will never form 6k -1 e 6k +1, the only ones that relate to the primes (but also semiprimes and powers of prime numbers), but fall into the other possible forms 6k, 6k + 2, 6k + 3, 6k + 4, as shown in the following table, for k = 0 and following, with an increase of one unit for each subsequent row. Prime numbers are marked in red, only 2 and 3 are in the forms 6k -1 e 6 k +1 being the ringleaders of thermultiples of 2 and 3

TABLE 1

6k – 4 Equivalente a 6k +2	6k – 3 Multipli dispari di 3	6k - 2	6k - 1	6k Multipli pari di 3	6k+1
2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31
32	33	34	35	36	37
38	39	40	41	42	43
44	45	46	47	48	49

50	**51**	**52**	**53**	**54**	**55**
56	**57**	**58**	**59**	**60**	**61**

(the numbers in blue are powers of 2: those with odd n are in column 6k - 4, equivalent to 6k + 2 (for example, 8 = 6 + 2 and 32 = 6 * 5 + 2, and those equal in column 6k - 2, for example. 4 = 6 - 2 and 16 = 6 * 3-2 = 18-2

This is important, as we will see below in the appropriate tables.

Indeed an odd power of 2 multiplied by an odd number of form 6k-1, gives a result that falls in the form 6k-2, and adding 1 to this result, we proceed to form 6k-1, and then possible prime number (eg. 8 * 5 = 40 and 40 +1 = 41 = 6 * 7 -1 = prime number).

If instead we multiply by an odd number of form 6k +1, the result falls in the column 6k - 4, and if we add 1, it falls in the form 6k-3 of odd multiples of 3, and then the number odd n form 6k +1 can be a number of Sierpinski for odd powers of 2, as we will see = eg. 8 * 7 = 56 and 56 +1 = 57 = 3 * 19 composite, such as all the numbers of the form 6k -3, except the 3 initial.

Prove that all the results of the formulas for the Sierpinski and Riesel numbers are all composite in the presence of k Sierpinski or Riesel number (not to be confused with the k of the forms of the numbers of above table), is equivalent to show that these results do not fall never in columns 6k -1 e 6k +1, or if we get (Sierpinski without the distinction

between even and odd powers of 2), are missing never a prime number, but only some of the composite semiprimes (or products of more prime numbers) or powers of prime numbers.

Demonstrated this, it is also shown that the respective formulas give only composite numbers. Let's go now to the related problems such as:

Which are the smaller Sierpinski and Riesel numbers?

The formulas do not distinguish between even powers of 2 (ie, n even exponent) and odd powers (with n odd).

With this our distinction, we find, with the following tables, the smaller Sierpinski and Riesel numbers are 5 and 7, followed by odd numbers of form 6k -1 and 6k+1). Then back on the general case, that is, for all powers of 2, no such distinction.

Tables with even *n* and *k =5* (or in the form *k* =6k - 1*)* for Sierpinsky:

TABLE 2 Sierpinsky:

n pari	**2^n**	**5 *2^n +1**	**Composti = Multipli di 3**
2	**4**	**21**	**3*7**
4	**16**	**81**	**3^4**
6	**64**	**321**	**3*107**
8	**256**	**1 281**	**3*7*61**
10	**1 024**	**5 121**	**3*3*569**
12	**4 096**	**20 481**	**3*6827**
…	**…**	**…**	**…**

As we see, all the results obtained are multiples of 3, and then 5 (but also all the other numbers of the form 6k -1, such as 11, 17, 23 etc. ..in fact 11 * 4 +1 = 45 = 3 ^ 3 * 5; 11 * 16 +1 = 177 = 3 * 59), and so on).

They are Sierpinski numbers, as they give all composite numbers as all divisible by 3 with the formula k * 2 ^ n +1, with n even, and obviously in this case the smallest is 5, which is also of the form 6k-1 = 6 * 1 – 1 = 6 - 1 = 5.

With n odd, then 5 * 2 ^ n and k = 5 instead we have always prime numbers in the last column with the results of 5 ^ 2 ^ n +1, and then 5 cannot now be Sierpinski number for odd powers of 2.

TABLE 3 Sierpinski

n dispari	2^n	5 *2^n +1	Primi e composti
1	2	5*2 + 1 =11	11 primo
3	8	5*8 + 1=41	41 primo
5	32	5*32 +1=	161 =7*23
7	128	5*128 + 1	129 =3*43
…	…	…	…

The odd powers of 2, then with the formula k * 2 ^ n +1 with odd n gives final results prime numbers and composite numbers, and so for them does not exist a number, however large or small, of Sierpinski.

Now let's see the odd powers of 2 with numbers of the form 6k +1, as the initial number 7:

TABELLA 4

n dispari	2^n	7 *2^n +1	Composti =Multipli di 3
1	2	7*2 + 1 =15	15=3*5
3	8	7* 8 + 1= 57	57=3*19
5	32	7*32 +1= 225	225=3^2*5^2
7	128	7*128 + 1=897	897=3*13*23
…	…	…	…

Now the composite numbers are of the form 7 * 2 ^ n +1 with n odd, and then k = 7 is their smallest Sierpinski number, such as 5 is the smallest for even powers of 2. If k = 13 = 6k +1, 6 +1 = 7 as, we have here all multiples of 3: only one example for

13 * 2 ^ 3 +1 +1 = 13 * 8 = 105 = 3 * 5 * 7

13 is therefore a Sierpinski number for odd powers of 2, but not the smallest (that is 7), as well as all subsequent numbers k odd of the form 6k + 1).

TABELLA 4

n pari	2^n	7 *2^n +1	Primi e composti
2	4	7*4 + 1 =29	primo
4	16	7* 16 +1= 113	primo
6	64	7*64 +1= 449	primo
8	256	7*256 + 1= 1793	11*163
…	…	…	…

The numbers of the form (6k +1) are therefore not Sierpinski numbers for even powers of 2,

Now to the Riesel numbers, for which the opposite is 5 for odd powers and 7 and for even powers equal to 2

TABLE 2.1

n pari	2^n	5 *2^n -1	Primi o no
2	4	19	19 primo
4	16	79	79 primo
6	64	319	319 =11*29
8	256	1 279	1 279 primo
10	1 024	5 119	5 119 primo
12	4 096	20 479	20 479 primo
…	…	…	…

Then 5 cannot be a Riesel number for even powers of 2, while it is for Sierpinski, see TABLE 2

TABELLA 3.1

n dispari	**2^n**	**5 *2^n -1**	**Multipli di 3**
1	**2**	**5*2 - 1 =9**	**9=3^2**
3	**8**	**5*8 - 1=39**	**39=3*13**
5	**32**	**5*32 -1=159**	**159=3*53**
7	**128**	**5*128 - 639**	**639=213**
…	**…**	**…**	**…**

Now 5 is a Riesel number, and precisely the smallest (the subsequent are all odd numbers of the form 6k-1, and therefore also prime numbers of this form, but also composite, eg. 35: In fact, 8 * 35-1 = 279 = 3 * 3 * 31 = multiple of 3)

In summary:

2 ^ n with even n, Sierpinski numbers of the form (6k-1), Sierpinski number minor = 5 = 6 * 1-1 = 6-1 = 5 because 5 * 2 ^ n +1 originates all

composite numbers and numbers of 3, as well as (6k-1) * 2 ^ n +1 generates only composite numbers and multiples of 3 (Table 2)

2 * n with odd n, the numbers of the form (6k-1) are not Sierpinski numbers, since (6k-1) * 2 ^ n +1 generates prime and composite number.

For Riesel numbers, instead we have:

for 2 ^ n with even n, the numbers of the form (6n-1) are not Riesel numbers, as (6k-1) * 2 ^ n-1 are not numbers of Riesel, because the formula generates prime and composite numbers numbers. (Table 3).

For 2 ^ n with n odd, the numbers of the form (6k-1) are Riesel numbers because the formula (6k-1) * 2 ^ n with n odd numbers generates all composite and multiples of 3 (Table 3.1) Here, too, the smallest Riesel number is 5.

For the form k = (6k +1), instead, they are Serpinski numbers for power of 2

It happens as for the numbers of Cullen and Woodall (Ref.1) but here we were looking for prime numbers (respectively Cullen and Woodall), now we seek only the composite numbers.

The definitions of Wikipedia about Sierpinski and Riesel numbers, do not, however, make distinctions between powers of 2 even or odd, and therefore such numbers must be valid for both powers of 2.

A example with the supposed first Riesel number (509203) for both powers of 2

TABLE 6

509203	509203*2^n -1 Con n pari e dispari	risultato	composto
n dispari 1	509203*2 -1	1018405 Di forma 6k +1	5*353*577
n pari 2	509203*4 -1	2036811 Di forma 6k-3 e quindi multiplo di 3	3*7*23*4217
n dispari 3	509203*8 -1	4073623 Di forma 6k +1	241*16903
n pari 4	509203*16 -1	8147247 Di forma 6k -3 e	3*2715749

		quindi multiplo di 3	
n dispari 5	**509203*32-1**	**16294495**	**5*7*19*107*229**
n pari 6	**509203 *64 -1**	**32588991**	**3^2*3620999**
n dispari 7	**509203*128-1**	**65177983**	**13*17*294923**
n pari 8	**509203*256-1**	**130355967**	**3*7*6207427**
n dispari 9	**509203*512 -1**	**260711935**	**5*11*4540217**
…	**…**	**…**	**…**

And here we return to the definition of Wikipedia, without distinction of odd or even *n*.

Iin these cases, ie without our distinction in even or odd powers of 2, the Sierpinski and Riesel numbers are those already known and reported from Wikipedia or other work.

As regards the forms 6k + 1, note that 509203 is form 6k -1 (in fact (509203 -1) / 6 = 84867, and for k of the form 6k – 1 (not to be confused k of k * 2 ^ n-1) with k 6k - 1

And for the forms 6k-1 they are Sierpinski numbers for even powers of 2, that is, with even n.

In fact, in Table 6, all the results for the even powers of 2 are divisible by 3, and then composite. The results for the odd powers also are all composite, but without the factor 3 common to the results for n even.

We note, however, that their factors are all of the form 6k -1 or all of the form 6k +1, for example 5, 353 and 577 are all of form 6k-1 (in fact 5 * 6-1, 353 = 354 -1 and 354/6 = 59, 577 = 588-1 and 588/6 = 98; for even powers we have multiple of 3 and for this reason 509203 that, of the form 6k - 1, is a Sierpinski number.

But for the odd powers, 509203 it is not a Sierpinski number.

Then the result could also be a prime number, yet the are all composite, but all of the form 6k + 1.

Some (one every 4), end with the digit 5, and then they are composite divisible by 5.

Others are multiples of 7, 11, 13, etc., maybe with automatisms similar to that for the factor of 5, so that for odd powers will always have equally composite numbers, as for even powers. And so for others Sierpinski numbers and other Riesel numbers.

We note that in the Table 6, where we have: **509203 *64 -1,** there is the number 64 ($64 = 8^2$) that is connected with the "modes" that correspond to the physical vibrations of a superstring by the following Ramanujan function:

$$8 = \frac{1}{3} \frac{4\left[anti\log \dfrac{\int_0^\infty \dfrac{\cos \pi t x w'}{\cosh \pi x} e^{-\pi x^2 w'} dx}{e^{-\frac{\pi t^2}{4} w'} \phi_{w'}(itw')} \right] \cdot \dfrac{\sqrt{142}}{t^2 w'}}{\log\left[\sqrt{\left(\dfrac{10 + 11\sqrt{2}}{4} \right)} + \sqrt{\left(\dfrac{10 + 7\sqrt{2}}{4} \right)} \right]}.$$

Conclusions

With our distinction of powers of 2 in odd or even of the exponent n (ie whether it is even or odd) we showed how to them, in the formula k * 2 ^ n-1 for the Sierpinski numbers k*2^n +1 for Riesel numbers, the results are alternately all composite and multiple of 3, or mixed between the prime and composite, and smaller Sierpinski and Riesel numbers are 5 and 7 and not big numbers like 509203
Without the above distinction, for Sierpinski (and therefore likewise to Riesel) powers like 2 give results all multiples of 3, and then all composite, while the odd powers give also results all composite, but multiples of 5, 7, 11, 13 etc.. with apparent irregularities (the only one that we saw in it the repetition factor of 5 for every four odd powers of 2).
With this work, and with our distinction for even or odd *n*, now we know a little 'more about the Sierpinski and Riesel numbers.

In conclusion, we want search a possible connection of the Sierpinski and Riesel numbers with the string theory.

Sierpinski's numbers

$78557 = 496 * 158 + 2 * 64 + 2 * 24 + 12 + 1$
$271129 = 496 * 513 + 64^2 + 24^2 * 16 + 64 * 48 + 27 * 11$
$271577 = 496 * 513 + 64^2 + 24^2 * 16 + 64 * 56 + 233$
$322523 = 496 * 513 + 64^2 * 14 + 24^2 * 18 + 11^2 * 3$
$327739 = 496 * 513 + 64^2 * 16 + 24^2 * 13 + 89 * 3$
$482719 = 496 * 513 + 64^2 * 48 + 24^2 * 48 + 5 * 11 * 73$
$575041 = 496 * 1026 + 64^2 * 12 + 24^3 + 3169$
$603713 = 496 * 1026 + 64^2 * 20 + 24^2 * 20 + 9^2 * 17$
$903983 = 496 * 1026 + 64^2 * 48 + 64^2 * 48 + 1871$

934909 = 496 * 1026 + 64^2 * 48 + 64^2 * 48 + 24^2 * 48 + 24^2 * 8 + 541
………

Riesel number

509203 = 496 * 1026 + 4 * 64 + 2 * 24 + 2 + 1
……….

Thence, decompositions where there are the number 8, 24 and 496 (or 12, 16, 64 and 48, where 12 = 24/2, 16 = 2 * 8, 64 = 8^2 and 48 = 2 * 24).
Considering the Sierpinski's number in this mode, we can obtain a mathematical connection between ALL these numbers and the modes corresponding to the physical vibrations of the superstrings and the bosonic strings (i.e. 8 and 24) by the following Ramanujan modular equations:

$$8 = \frac{1}{3}\frac{4\left[anti\log\frac{\int_0^\infty \frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2 w'}dx}{e^{-\frac{\pi t^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2 w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}. \quad (1)$$

$$24 = \frac{4\left[anti\log\frac{\int_0^\infty \frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2 w'}dx}{e^{-\frac{\pi t^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2 w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}. \quad (2)$$

Furthermore, we have that, for example:

271577 = 496 * 513 + 64^2 + 24^2 * 16 + 64 * 56 + 233 = 254448 + 4096 + + 9216 + 3584 + 233;
603713 = 496 * 1026 + 64^2 * 20 + 24^2 * 20 + 9^2 * 17 = 508896 + 81920 + + 11520 + 1377;

These numbers: 508896, 254448, 81920, 11520, 9216, 4096, 3584, 1377 and 233 and each number that we obtain from the other Sierpinski's numbers, can be considered all new solutions regarding the equations of the bosonic strings and superstrings theory

4. REFERENCES

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